Thermodynamic Modeling and Control of Screw Extruder for 3D Printing

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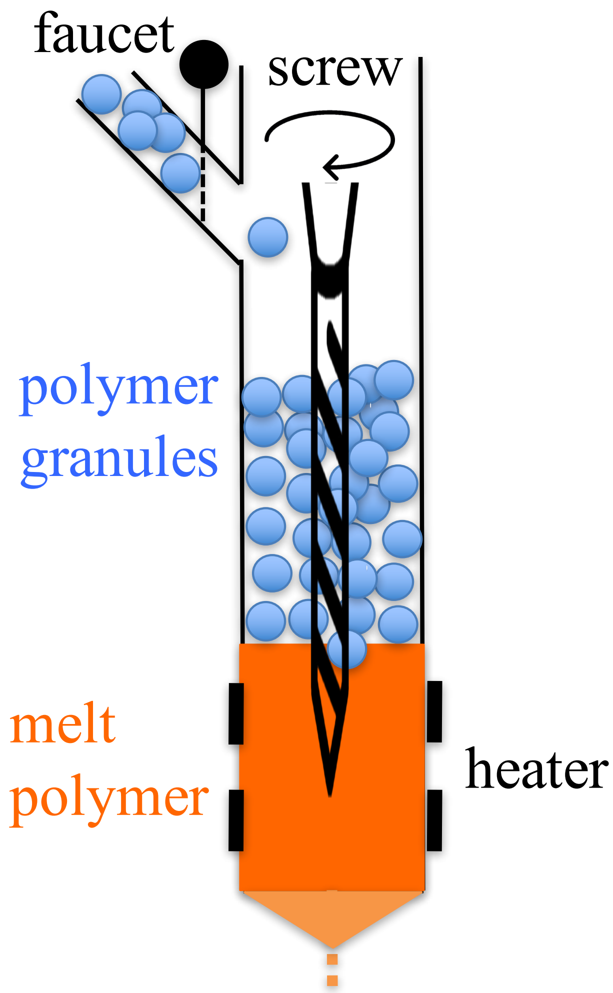
Schematic of Screw Extruder



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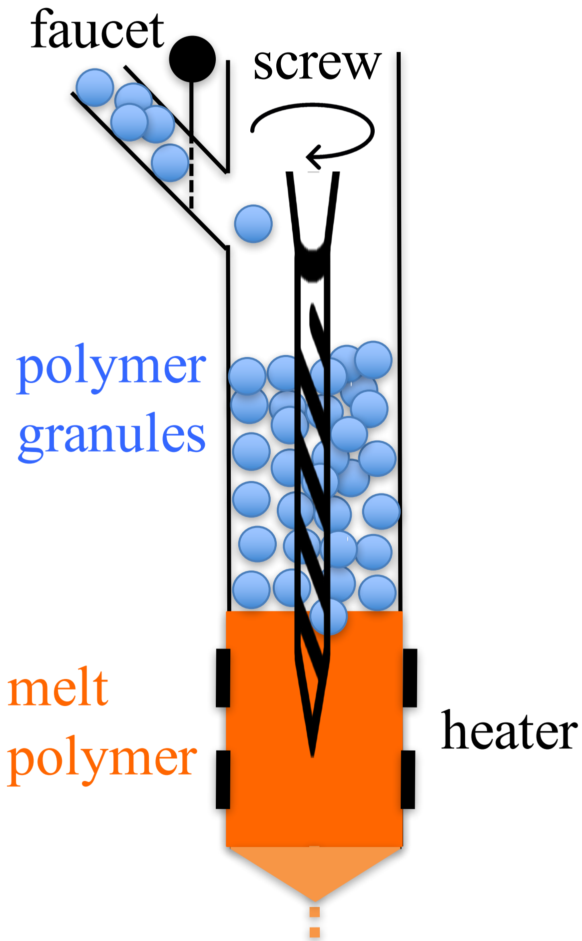

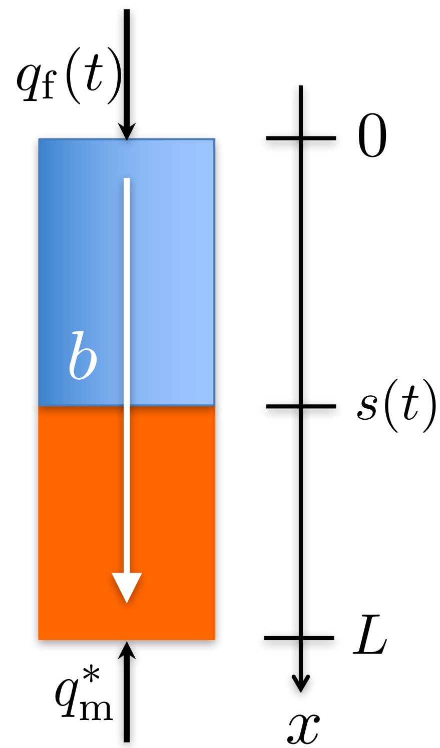

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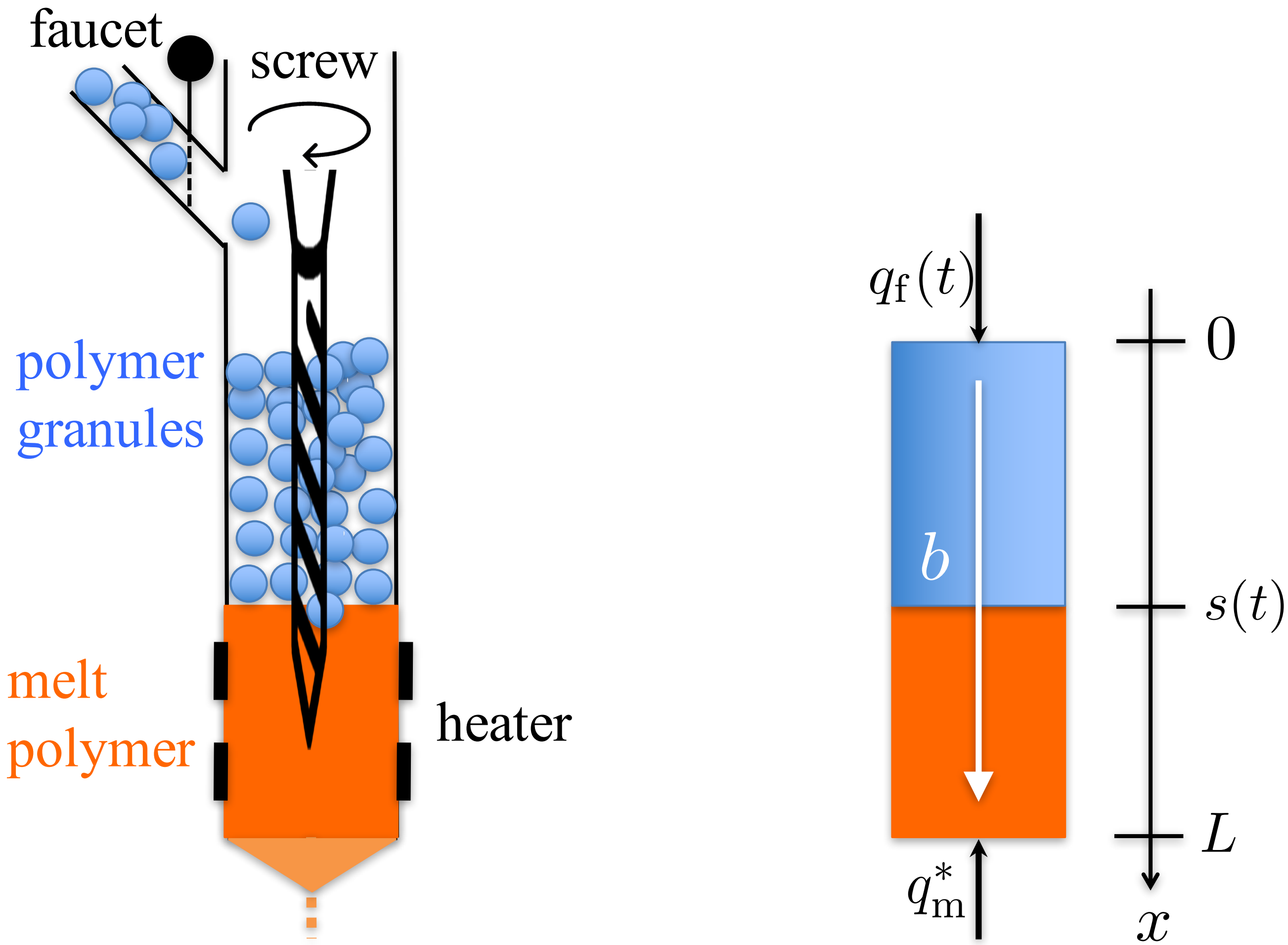
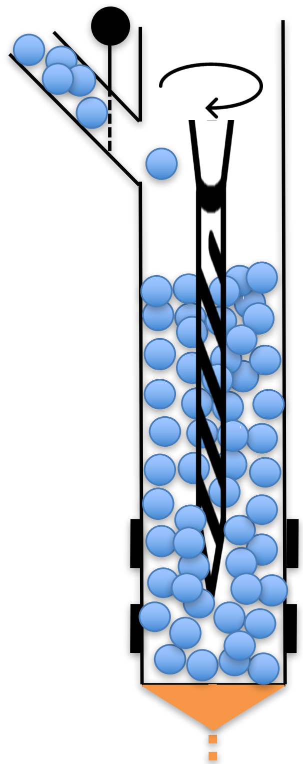
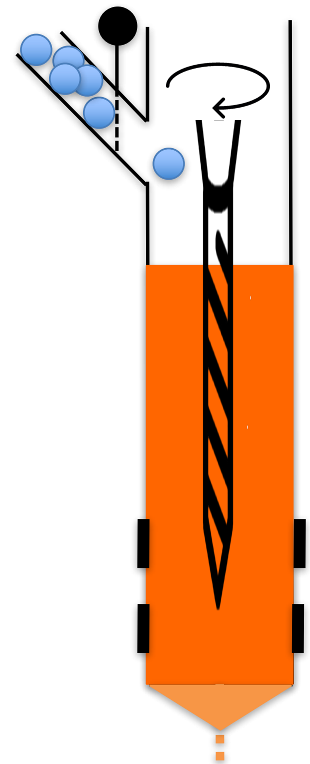


Chosen values : b (extrusion speed), q^* (outlet heater), T_b (barrel temp.),
Control : $q_{\mathrm{f}}(t)$ (inlet heat by mixing fed granules with different temperature)

Want to avoid

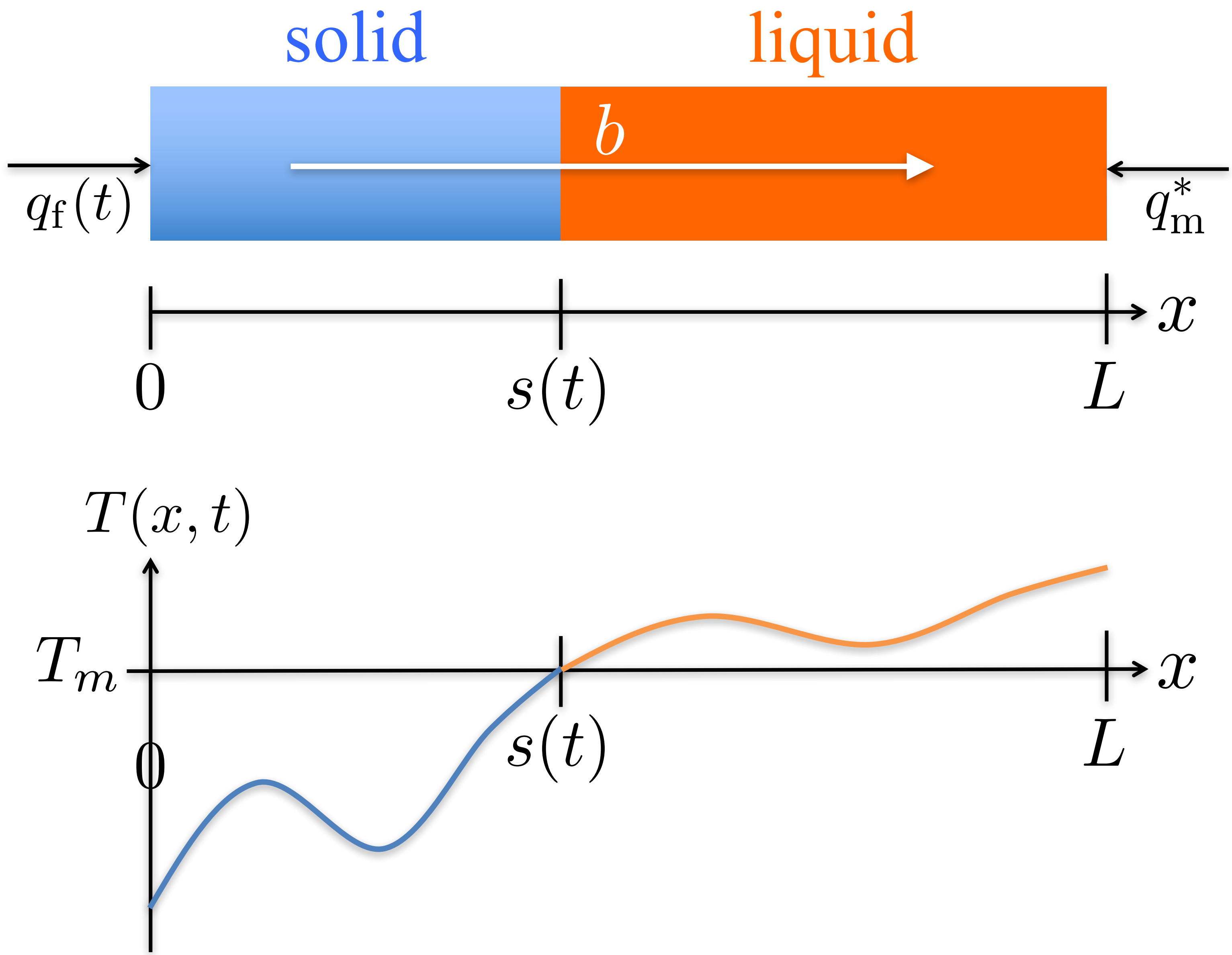


Ink run out



overheating

Problem : Stabilize ratio of granules/melt polymer



Goal : Design cooling heat $q_{\mathrm{f}}(t)$ to achieve $s(t) \to s_r$.

Thermodynamic Modelling

PDEs (solid & liquid)

$$\frac{\partial T_\mathsf{s}}{\partial t}(x,t) = \alpha_\mathsf{s}\frac{\partial^2 T_\mathsf{s}}{\partial x^2}(x,t) - b\frac{\partial T_\mathsf{s}}{\partial x}(x,t) + h_\mathsf{s}\left(T_\mathsf{b} - T_\mathsf{s}(x,t)\right), \text{for} \quad 0 < x < s(t),$$

$$\frac{\partial T_\mathsf{l}}{\partial t}(x,t) = \alpha_\mathsf{l}\frac{\partial^2 T_\mathsf{l}}{\partial x^2}(x,t) - b\frac{\partial T_\mathsf{l}}{\partial x}(x,t) + h_\mathsf{l}\left(T_\mathsf{b} - T_\mathsf{l}(x,t)\right), \text{for} \quad s(t) < x < L,$$

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$$\frac{\partial T_\mathsf{l}}{\partial t}(x,t) = \alpha_\mathsf{l} \frac{\partial^2 T_\mathsf{l}}{\partial x^2}(x,t) - b\frac{\partial T_\mathsf{l}}{\partial x}(x,t) + h_\mathsf{l}\left(T_\mathsf{b} - T_\mathsf{l}(x,t)\right), \text{for} \quad s(t) < x < L,$$

BCs

$$\frac{\partial T_\mathsf{s}}{\partial x}(0,t) = -\frac{q_\mathsf{f}(t)}{k_\mathsf{s}}, \quad \frac{\partial T_\mathsf{l}}{\partial x}(L,t) = \frac{q^*_\mathsf{m}}{k_\mathsf{l}}$$

$$T_\mathsf{s}(s(t),t) = T_\mathsf{l}(s(t),t) = T_\mathsf{m},$$

Thermodynamic Modelling

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$$T_{\mathsf{s}}(s(t),t) = T_{\mathsf{l}}(s(t),t) = T_{\mathsf{m}},$$

ODE (Interface dynamics)

$$\dot{s}(t) = \bar{\beta}\left(k_{\mathsf{s}}\frac{\partial T_{\mathsf{s}}}{\partial x}(s(t),t) - k_{\mathsf{l}}\frac{\partial T_{\mathsf{l}}}{\partial x}(s(t),t)\right)$$

Model valid iff

$$T_s(x,t) \leq T_m, \quad \text{for} \quad \forall x \in (0, s(t)), \quad \forall t > 0$$

$$T_l(x,t) \geq T_m, \quad \text{for} \quad \forall x \in (s(t), L), \quad \forall t > 0$$

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Steady-State (SS)

$$\begin{cases} T_{\mathrm{l,eq}}(x) = & p_1 e^{q_1 (x - s_{\mathrm{r}})} + p_2 e^{q_2 (x - s_{\mathrm{r}})} + T_{\mathrm{b}}, \\ T_{\mathrm{s,eq}}(x) = & p_3 e^{q_3 (x - s_{\mathrm{r}})} + p_4 e^{q_4 (x - s_{\mathrm{r}})} + T_{\mathrm{b}}, \end{cases}$$

where p_i & q_i are parameters calculated by known physical values.

$\star$ If $T_b = T_m$ & $q_{\mathrm{m}}^* = 0$, then $T_{\mathrm{s,eq}}(x) = T_{\mathrm{l,eq}}(x) = T_{\mathrm{m}}$.

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$\star$ If $T_b = T_m$ & $q_{\mathrm{m}}^* = 0$, then $T_{\mathrm{s,eq}}(x) = T_{\mathrm{l,eq}}(x) = T_{\mathrm{m}}$.

Lemma : If the barrel temperature satisfies

$$T_{\mathrm{m}} - \underline{q} \le T_{\mathrm{b}} \le T_{\mathrm{m}} + \bar{q}$$

where $\underline{q}$ & $\bar{q}$ are parameters calculated by known physical values, then $T_{\mathrm{s,eq}}(x) \le T_m$ & $T_{\mathrm{l,eq}}(x) \ge T_m$.

Assumption : Liquid temperature is at SS, i.e., $T_l(x,t) = T_{\mathsf{l,eq}}(x)$

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Reference Error States

$$u(x,t) = k_{\mathsf{s}}(T_{\mathsf{s,eq}}(x) - T_{\mathsf{s}}(x,t)), \quad X(t) = s(t) - s_{\mathsf{r}}$$

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Reference Error System

$$\begin{aligned}
\frac{\partial u}{\partial t}(x,t) =& \alpha_{\mathrm{s}}\frac{\partial^2 u}{\partial x^2}(x,t) - b\frac{\partial u}{\partial x}(x,t) - h_{\mathrm{s}}u(x,t), \\
\frac{\partial u}{\partial x}(0,t) =& -U(t), \quad u(s(t),t) = k_{\mathrm{s}}(T_{\mathrm{s,eq}}(s(t)) - T_{\mathrm{m}}), \\
\dot{X}(t) =& -\bar{\beta}\frac{\partial u}{\partial x}(s(t),t) + \bar{\beta}\left(k_{\mathrm{s}}T'_{\mathrm{s,eq}}(s(t)) - k_{\mathrm{l}}T'_{\mathrm{l,eq}}(s(t))\right),
\end{aligned}$$

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\end{aligned}$$

Linearize blue eq. around $s(t) \approx s_r$.

Linearized Reference Error System

$$\begin{aligned}\frac{\partial u}{\partial t}(x,t) =& \alpha_{\mathsf{s}}\frac{\partial^2 u}{\partial x^2}(x,t) - b\frac{\partial u}{\partial x}(x,t) - h_{\mathsf{s}}u(x,t),\\ \frac{\partial u}{\partial x}(0,t) =& -U(t), \quad u(s(t),t) = CX(t),\\ \dot{X}(t) =& AX(t) - \bar{\beta}\frac{\partial u}{\partial x}(s(t),t),\end{aligned}$$

where C & A are obtained by known values, and $C > 0$.

Linearized Reference Error System

$$\frac{\partial u}{\partial t}(x,t) = \alpha_{\mathrm{S}} \frac{\partial^2 u}{\partial x^2}(x,t) - b\frac{\partial u}{\partial x}(x,t) - h_{\mathrm{S}} u(x,t),$$

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Backsteppinng Transformation

$$w(x,t) = u(x,t) - \frac{\bar{\beta}}{\alpha_{\mathrm{S}}} \int_x^{s(t)} \phi(x-y)u(y,t)dy - \phi(x - s(t))X(t),$$

Linearized Reference Error System

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Backsteppinng Transformation

$$w(x,t) = u(x,t) - \frac{\bar{\beta}}{\alpha_S}\int_x^{s(t)} \phi(x-y)u(y,t)dy - \phi(x-s(t))X(t),$$

Target System

$$\frac{\partial w}{\partial t}(x,t) = \alpha_S \frac{\partial^2 w}{\partial x^2}(x,t) - b\frac{\partial w}{\partial x}(x,t) - h_S w(x,t)$$
$$+ \dot{s}(t)\phi'(x-s(t))X(t), \quad 0 < x < s(t)$$
$$\frac{\partial w}{\partial x}(0,t) = \frac{b}{2\alpha_S}w(0,t), \quad w(s(t),t) = CX(t),$$
$$\dot{X}(t) = (A-c)X(t) - \bar{\beta}\frac{\partial w}{\partial x}(s(t),t),$$

Control law (derived by transformation & target system)

$$U(t) = -\frac{b}{2\alpha_{\mathsf{S}}}u(0,t) - \frac{\bar{\beta}}{\alpha_{\mathsf{S}}}\int_0^{s(t)} f(x)u(x,t)dx - f(s(t))X(t),$$

where $f(x) = \phi'(-x) - \gamma\phi(-x)$.

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Theorem : Under $T_b = T_m$ & $q_{\mathrm{m}}^* = 0$, the control law makes the closed-loop system exponentially stable in the norm $||T_{\mathrm{s}}(x,t) - T_{\mathrm{s,eq}}(x)||^2_{\mathcal{H}_1} + (s(t) - s_r)^2$

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Note : Under $T_b = T_m$ & $q_{\mathrm{m}}^* = 0$, the problem is equivalent to

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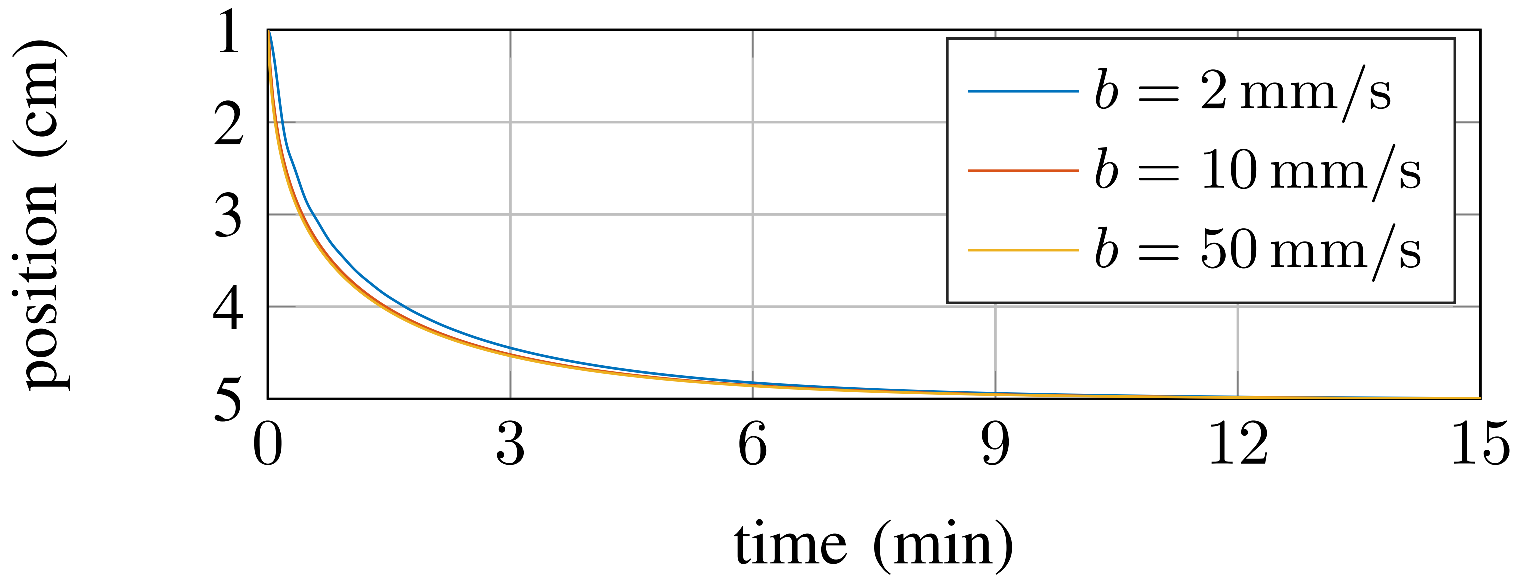
Apply the designed control to the *original two-phase model* in numerical simulation

Numerical Simulation

L = 10 [cm], $s_r = 5$ [cm],

T_b = 145 [°C], $T_m = 135$ [°C] , $q_m^* =$100 [W/m^2]



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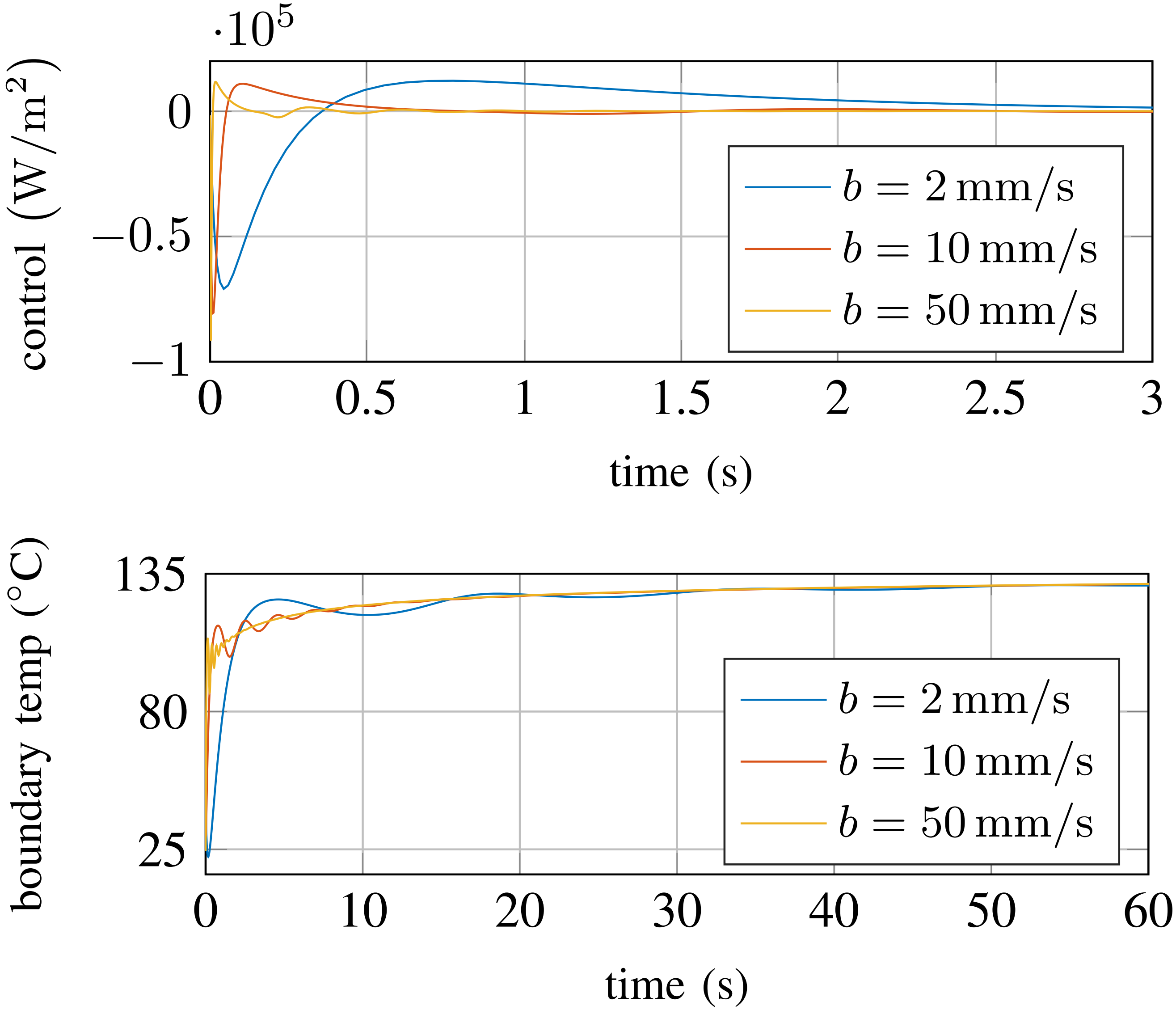

The boundary temperature remains a reasonable value

Future Work

- Observer-based output feedback control

- Relax assumption of $T_l(x,t) = T_{\mathrm{l,eq}}(x)$, and design for two-phase dynamics

- Experimental verification of thermodynamic model