Arctic Sea Ice Temperature Profile Estimation via Backstepping Observer Design

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CCTA 2017

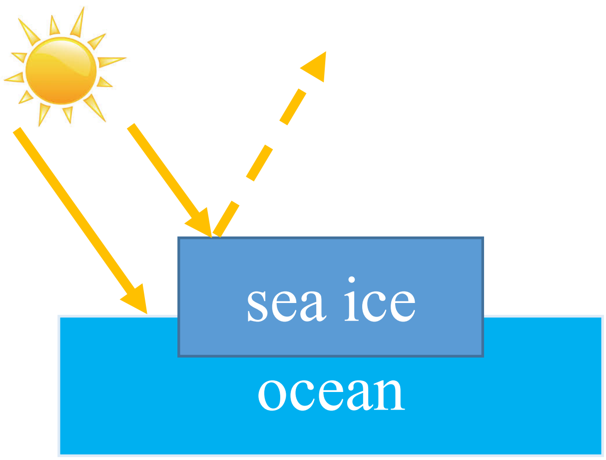
What Is Sea Ice?

- Sea ice is frozen ocean water.
  (while icebergs, glaciers, etc.
  originate in land)

- It covers 12% of the ocean.

Why Is Arctic Sea Ice Important?

- Affects global climate by reflection of solar energy.



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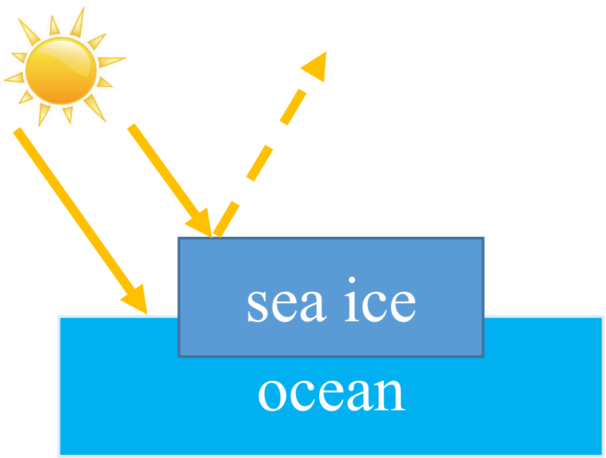
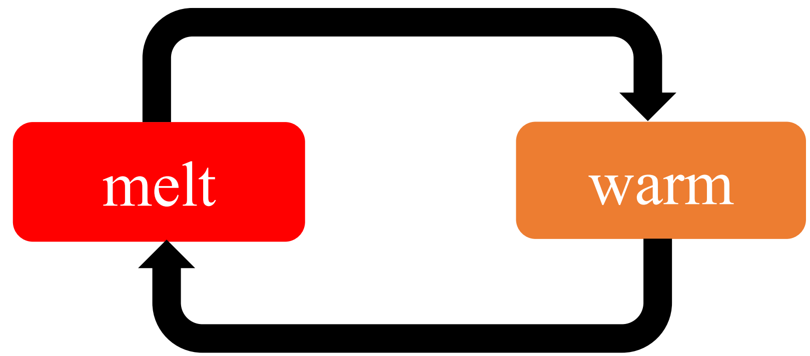
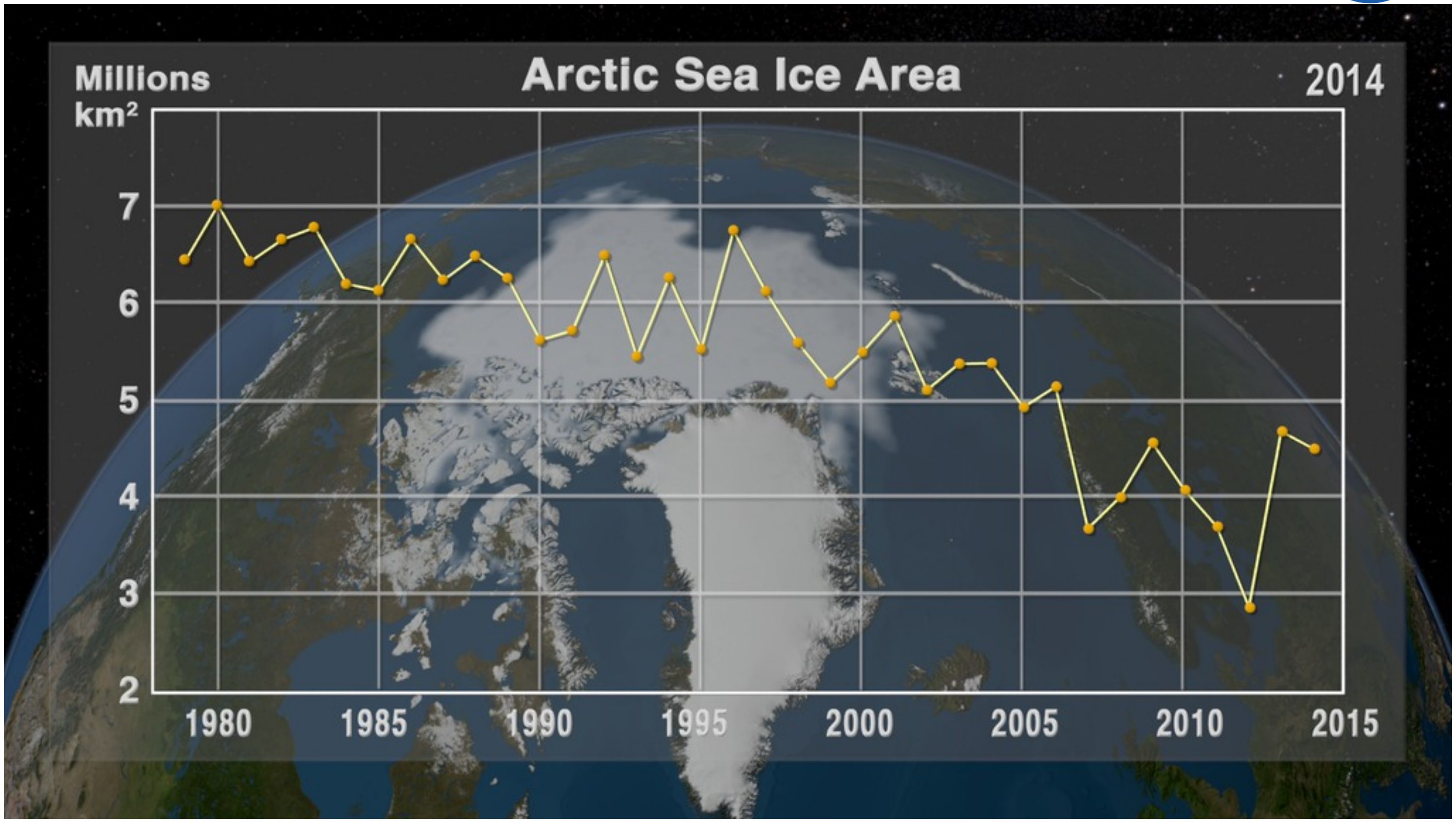




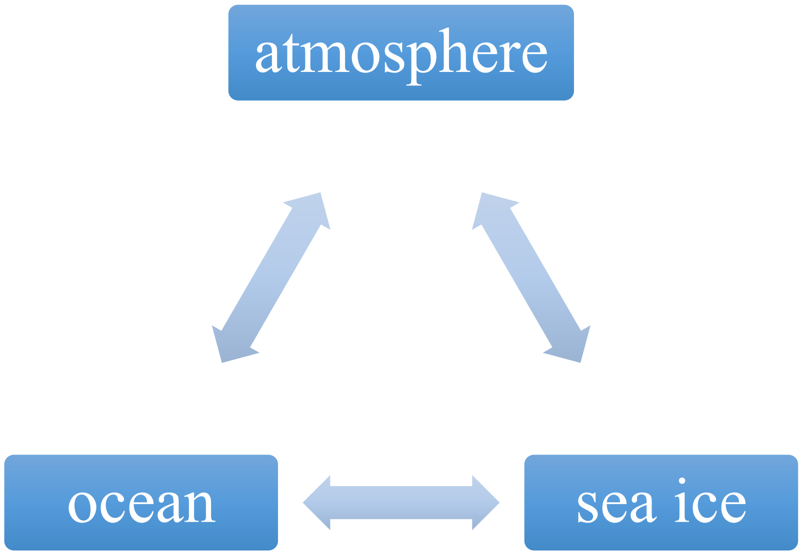
Ice-albedo positive feedback

Why Is Arctic Sea Ice Important?

- Recent decline of Arctic sea ice



Sea Ice in Global Climate Model



…… Global climate model

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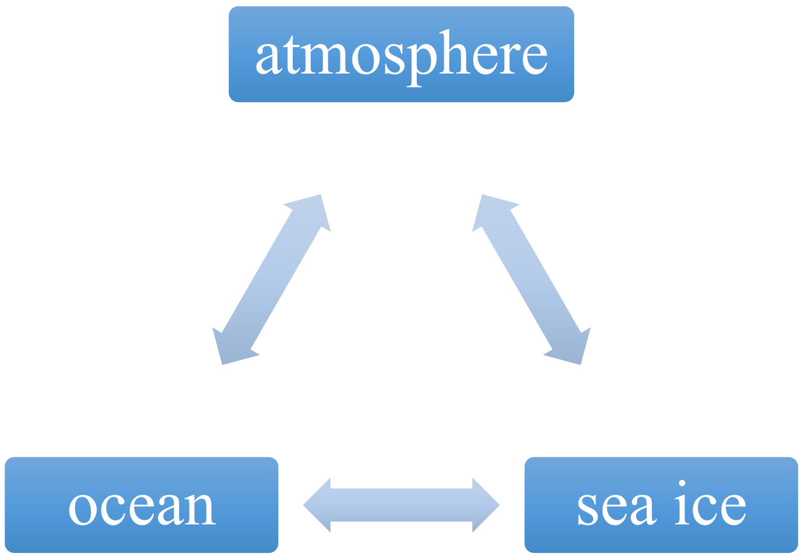

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- Time-evolution of thickness and temperature by Maykut and Untersteiner, 1971 (MU71)

- Time-evolution of thickness distribution by Thorndike, et al, 1975

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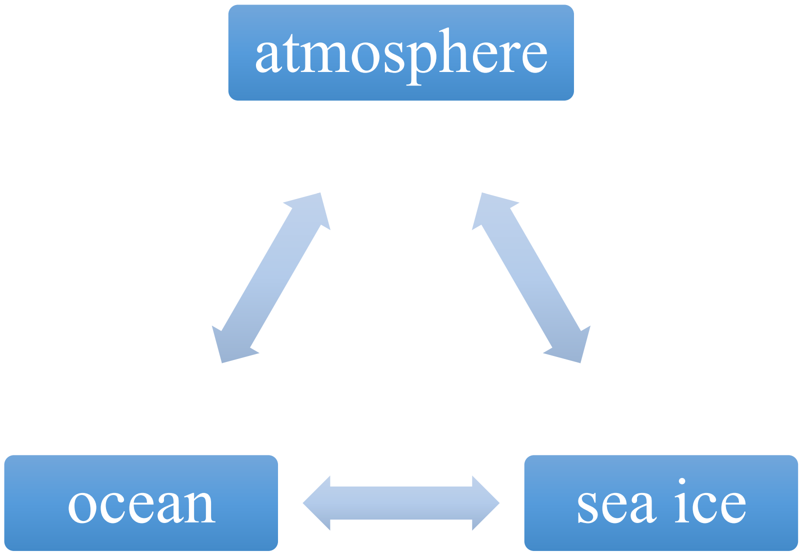

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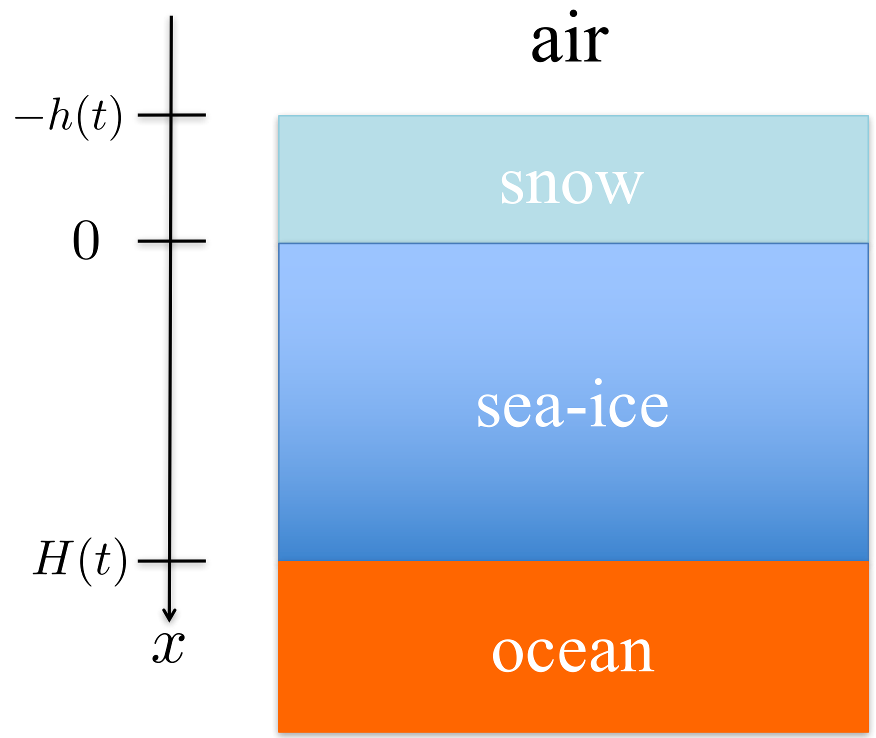
Thermodynamic Model by MU71

$(T_s(x,t), T_i(x,t)) \cdots$ Temp. of snow, sea ice



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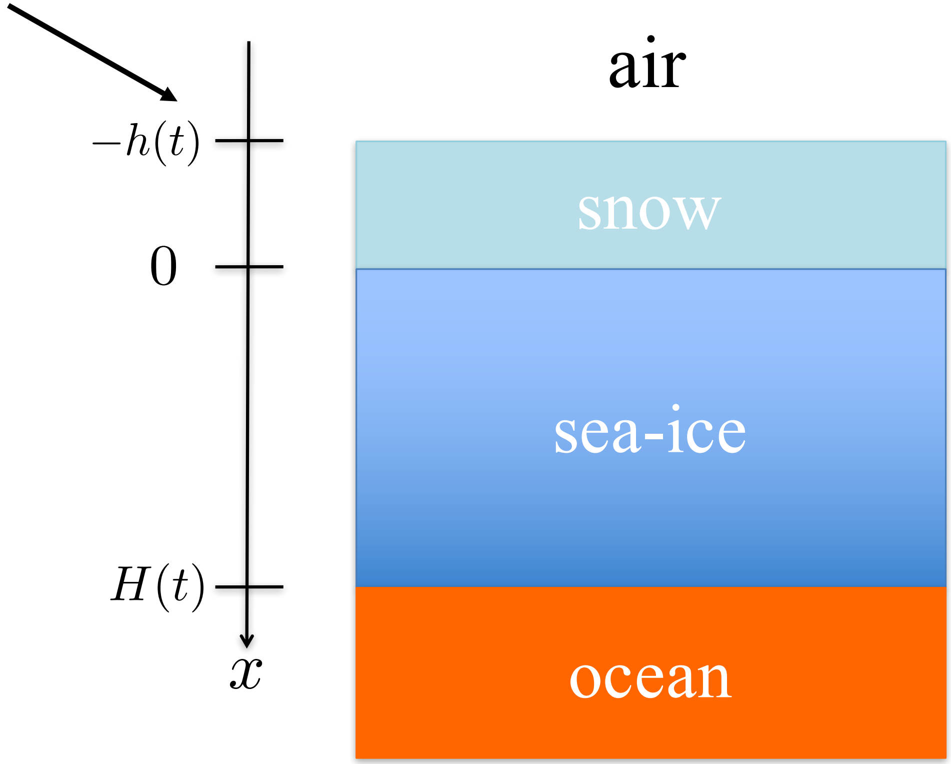
$$F_a - \sigma(T_s(-h(t),t)+273)^4 + k_s \frac{\partial T_s}{\partial x}(-h(t),t) = \begin{cases} 0, & \text{if} \quad T_s(-h(t),t) < T_{m1}, \\ -q\dot{h}(t), & \text{if} \quad T_s(-h(t),t) = T_{m1}, \end{cases}$$



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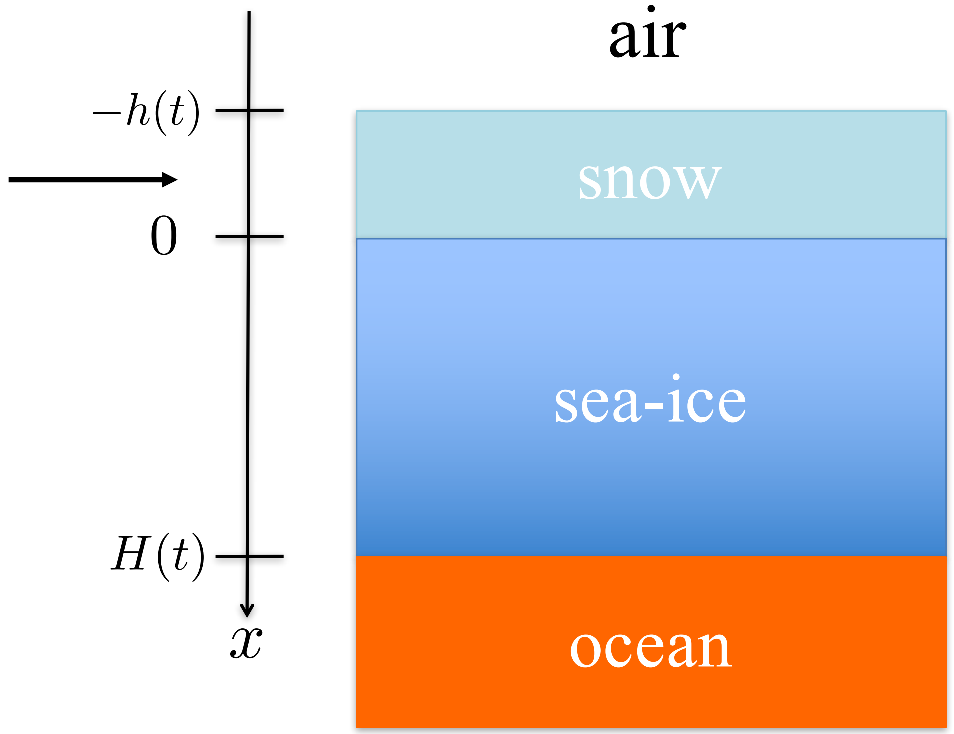

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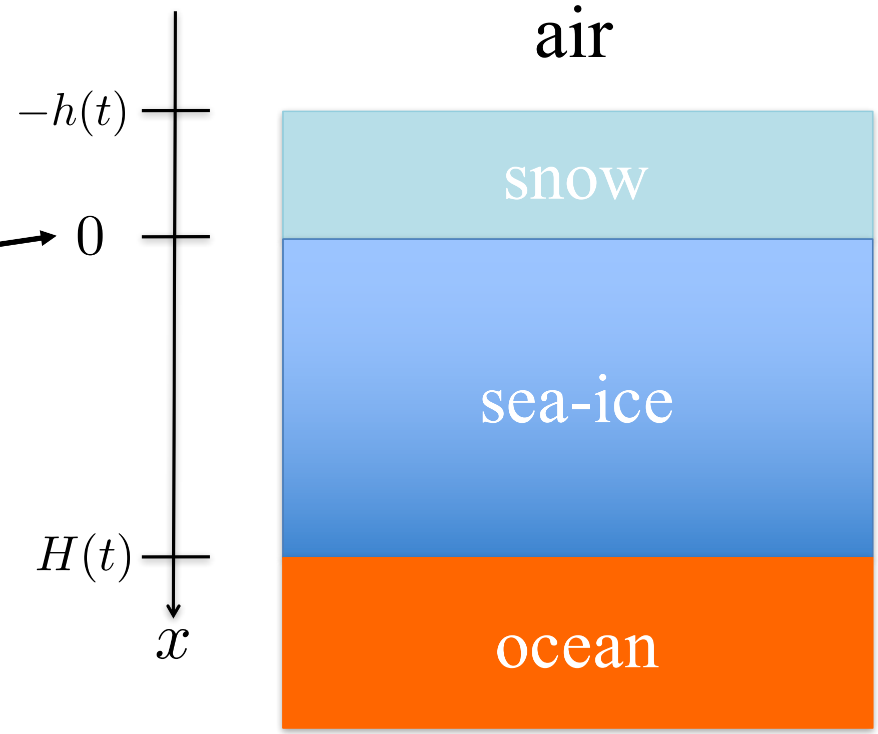

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&+ I_0 \kappa_i e^{-\kappa_i x}, \quad 0 < x < H(t),
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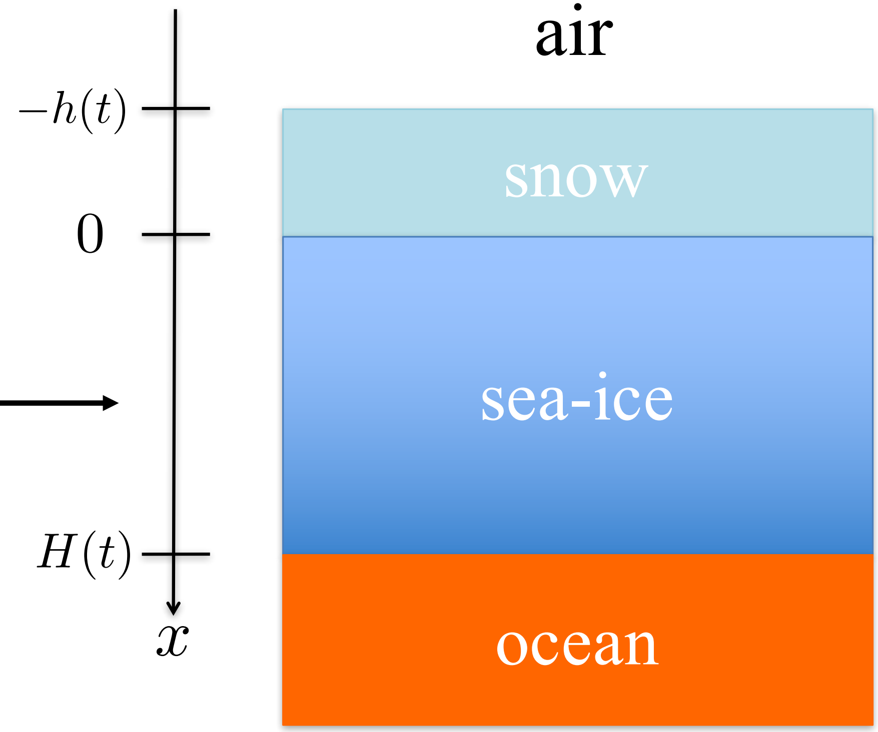

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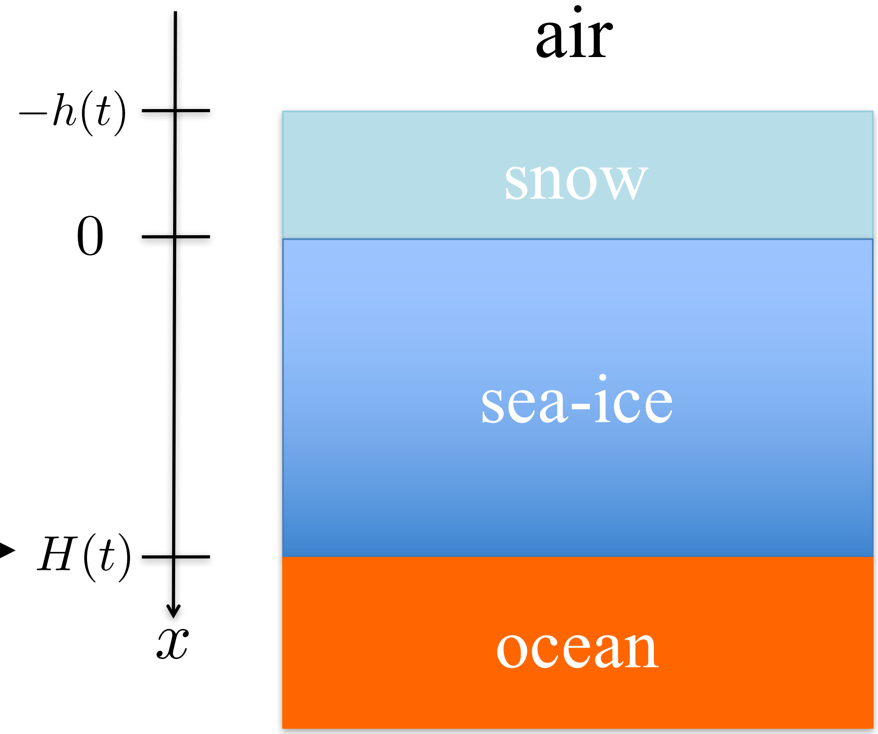
&\quad + I_0 \kappa_i e^{-\kappa_i x}, \quad 0 < x < H(t), \\
T_i(H(t),t) &= T_{m2}, \\
q\dot{H}(t) &= k_i \frac{\partial T_i}{\partial x}(H(t),t) - F_w,
\end{aligned}
$$



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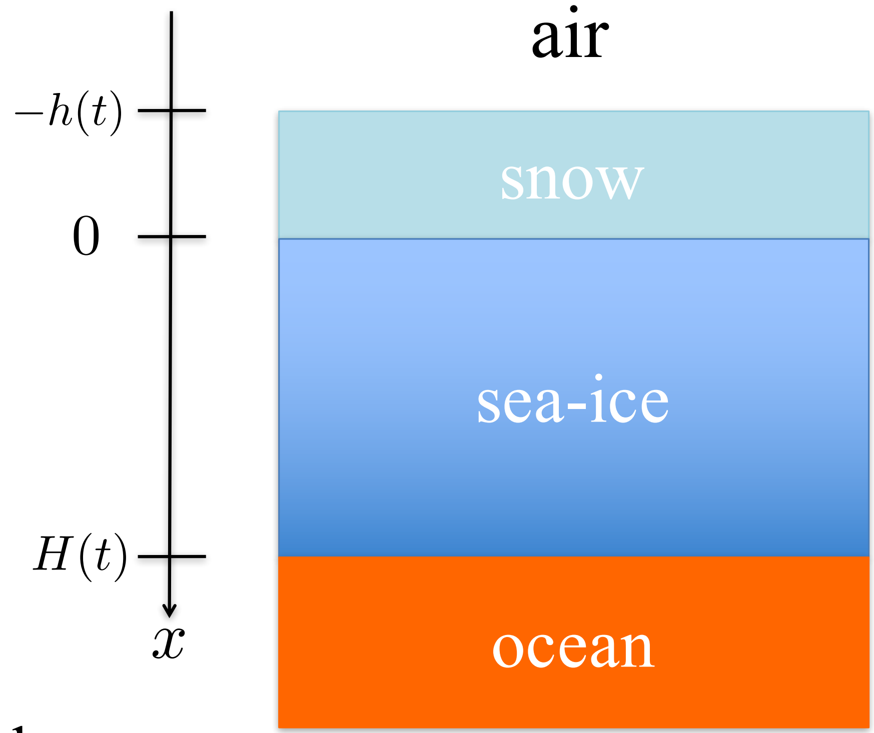


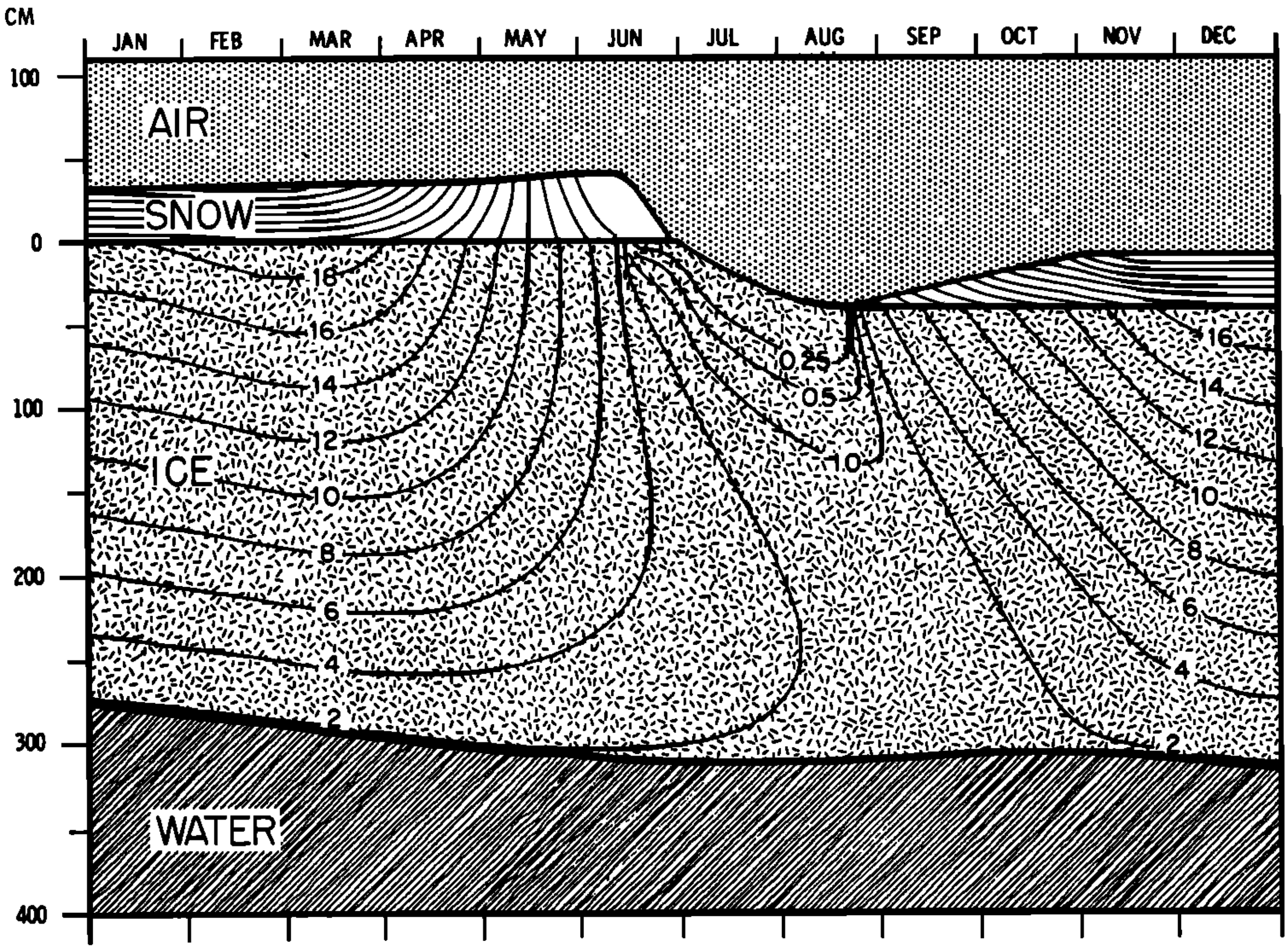
Salinity

$$S(x) = A\left[1 - \cos\left\{\pi\left(\frac{x}{H(t)}\right)^{\frac{n}{m+\frac{x}{H(t)}}}\right\}\right]$$

Dependence

$$
\begin{cases} c_i(T_i, S(x)) = c_0 + \gamma \frac{S(x)}{T_i(x,t)^2}, \\ k_i(T_i, S(x)) = k_0 + \beta \frac{S(x)}{T_i(x,t)} \end{cases}
$$

Simulation Result by MU71



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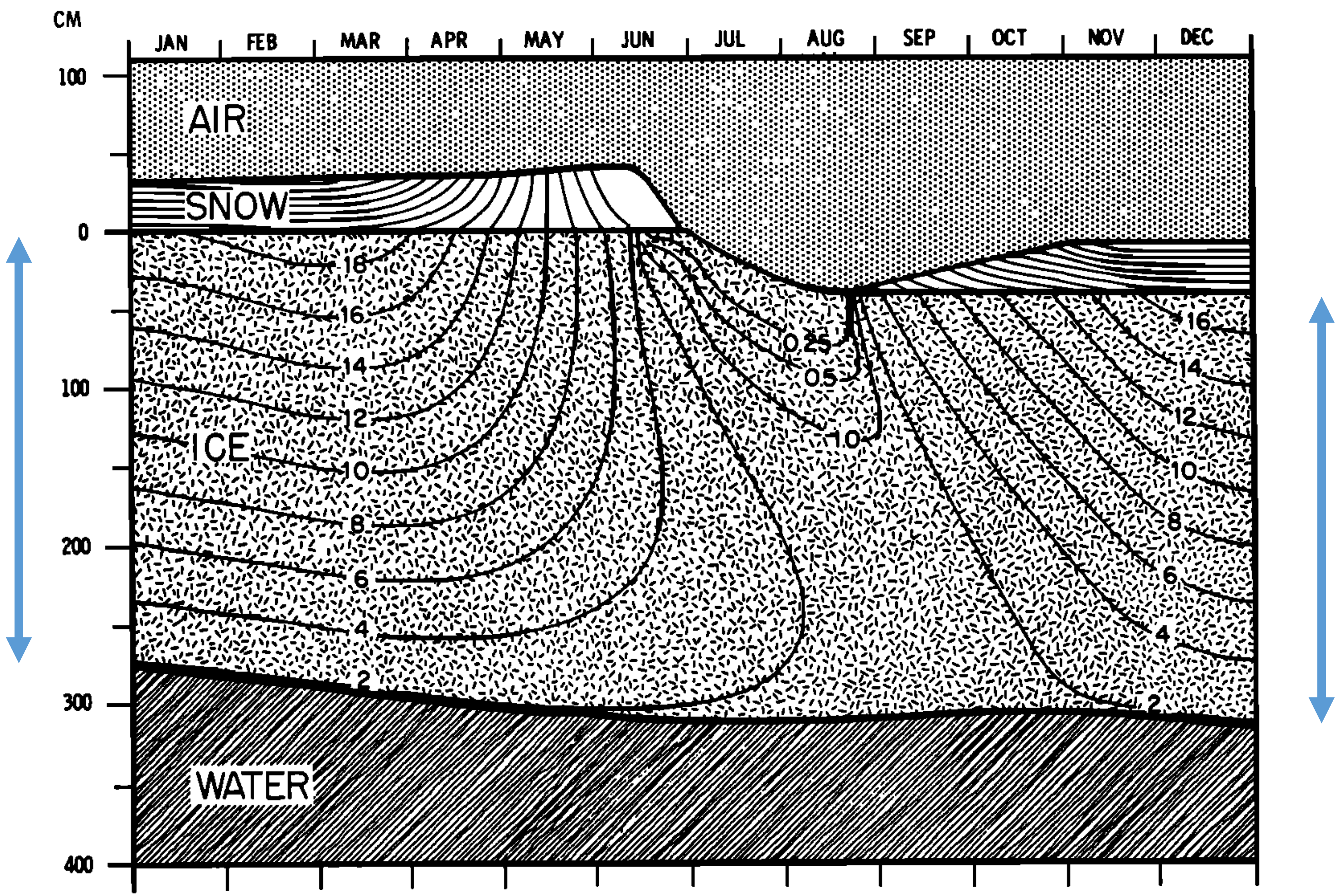
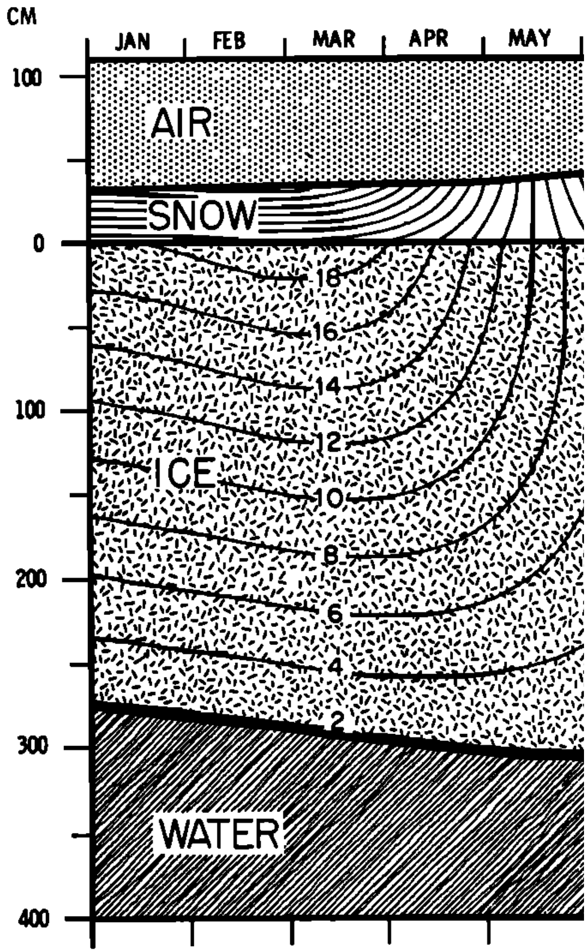
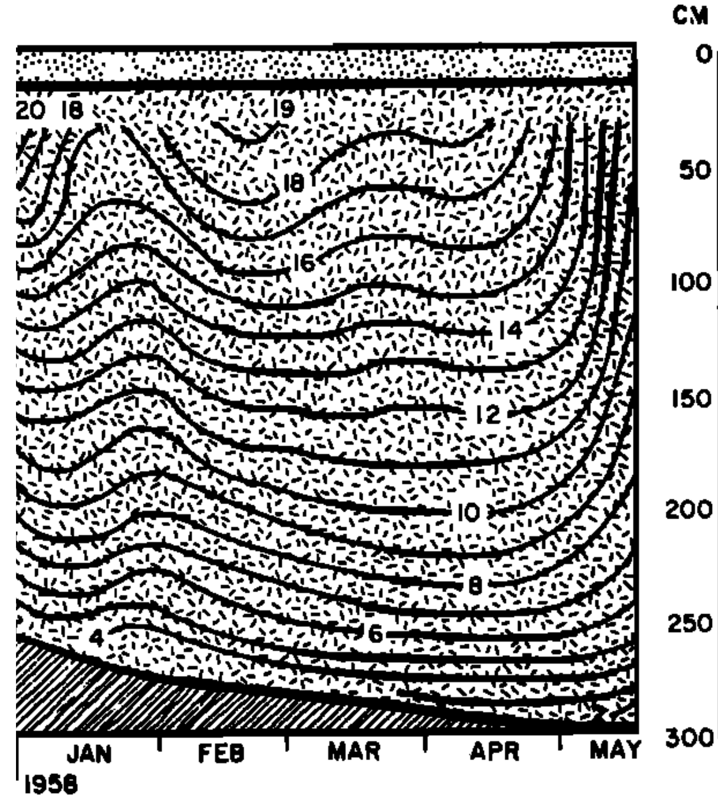


Comparison with Empirical Data

Simulation (MU71)



Empirical Data
(Untersteiner 1969)



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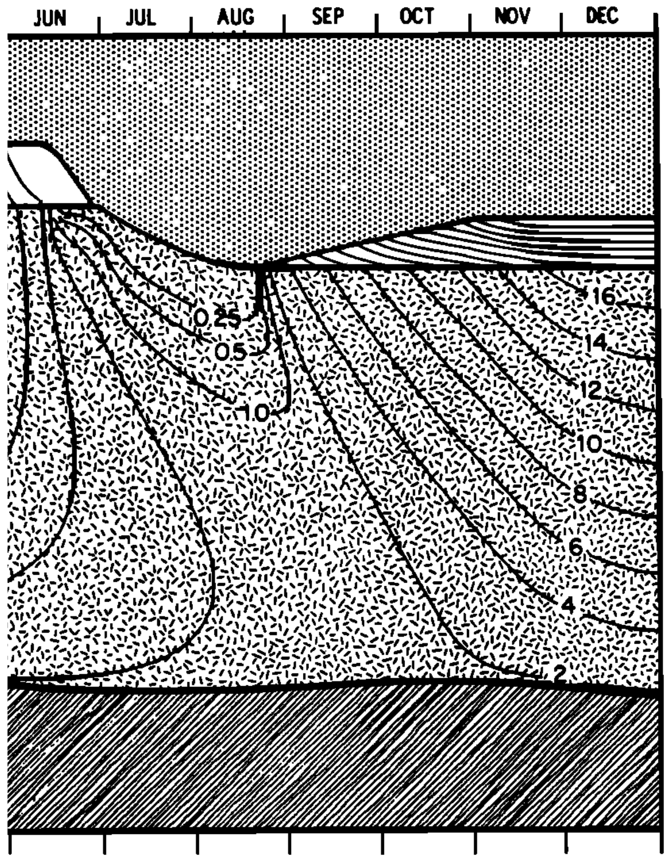

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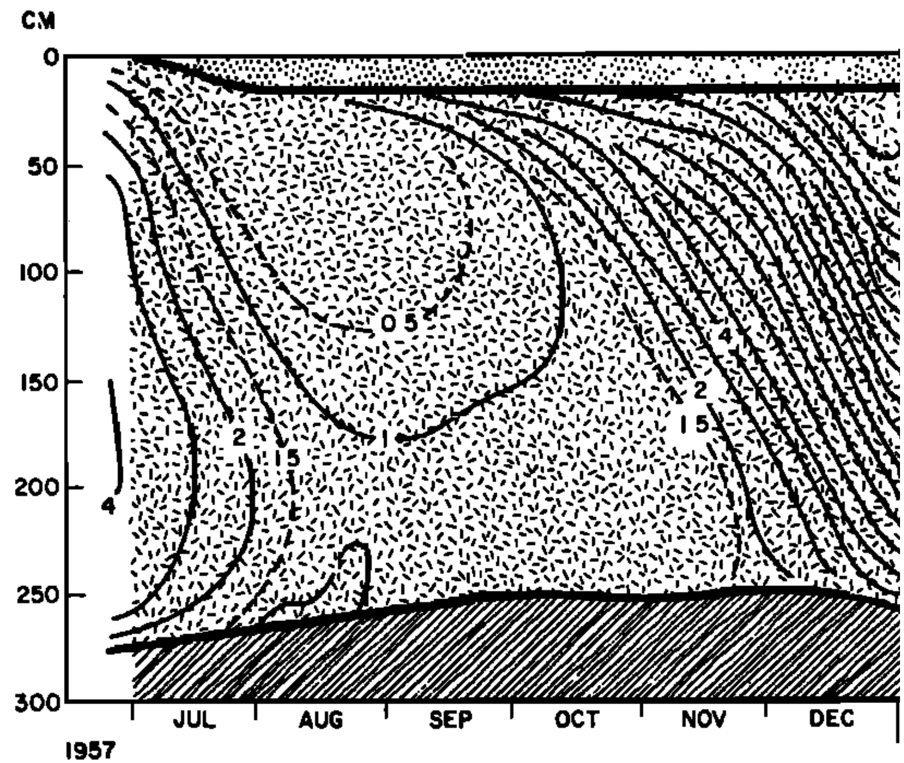

Problem Statement

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1) Recent data shows no annual cycle.
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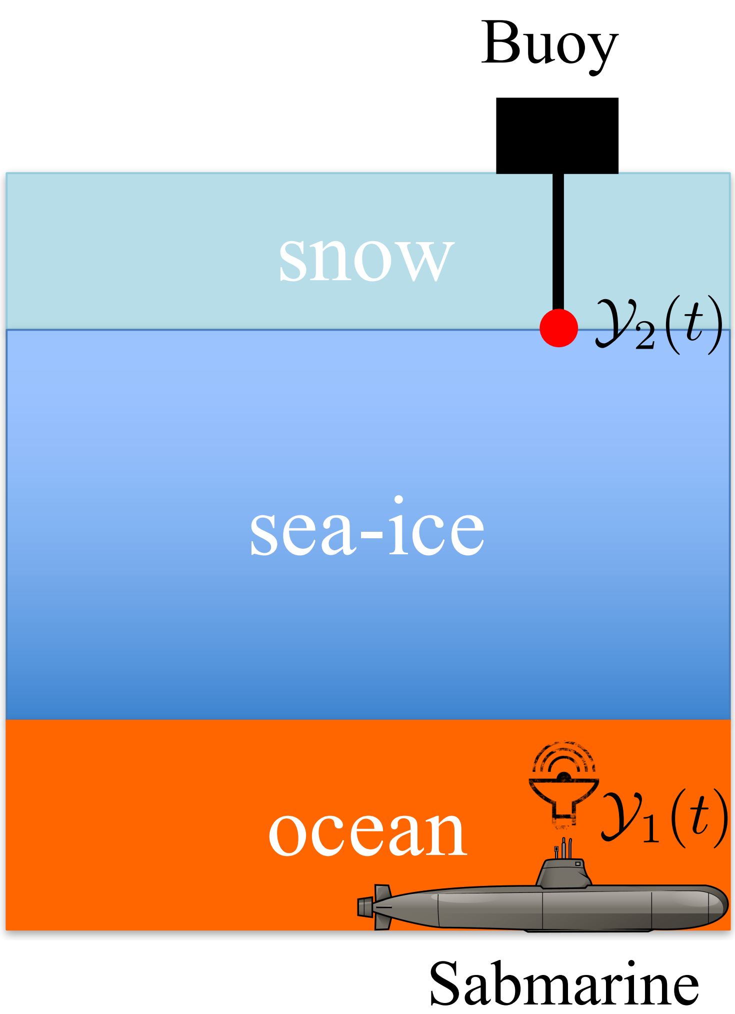
- **Method**

1) Design an estimator for simplified MU71 theoretically.
2) Apply the estimator to original MU71 numerically.

State Estimation via Backstepping Observer

- Available Measurements

$$\mathcal{Y}_1(t) = H(t),$$
$$\mathcal{Y}_2(t) = T_i(0,t),$$
$$\mathcal{Y}_3(t) = \frac{\partial T_i}{\partial x}(H(t),t).$$



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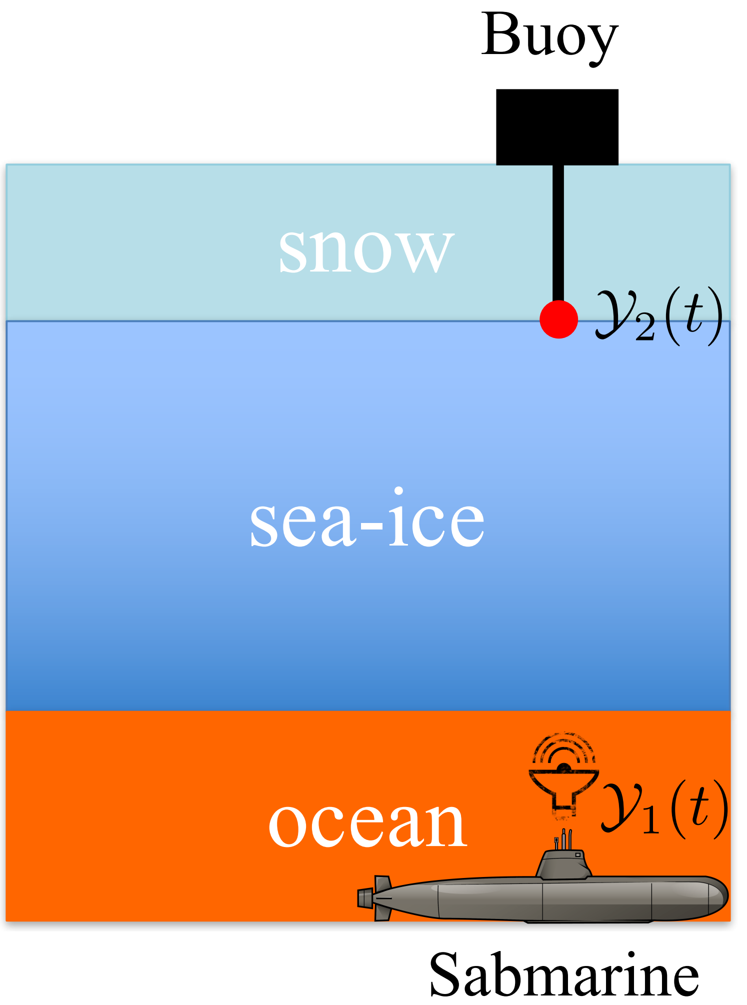
- Simplified MU71

Salinity free : $S(x) = 0$



State Estimation via Backstepping Observer

- Observer Design

$$\hat{T}_i(0,t) = \mathcal{Y}_2(t),$$

$$\frac{\partial \hat{T}_i}{\partial t}(x,t) = D_i \frac{\partial^2 \hat{T}_i}{\partial x^2}(x,t) + \bar{I}_0 \kappa_i e^{-\kappa_i x} + p(x,t)\left(\mathcal{Y}_3(t) - \frac{\partial \hat{T}_i}{\partial x}(\mathcal{Y}_1(t),t)\right), \quad 0 < x < \mathcal{Y}_1(t)$$

$$\hat{T}_i(\mathcal{Y}_1(t),t) = T_{m2}.$$

State Estimation via Backstepping Observer

- Observer Design

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\hat{T}_i(\mathcal{Y}_1(t),t) =& T_{m2}.
\end{aligned}
$$

- Error System

$$
\begin{aligned}
\tilde{T}_i(0,t) =& 0, \\
\frac{\partial \tilde{T}_i}{\partial t}(x,t) =& D_i \frac{\partial^2 \tilde{T}_i}{\partial x^2}(x,t) - p(x,t)\frac{\partial \tilde{T}_i}{\partial x}(H(t),t), \\
\tilde{T}_i(H(t),t) =& 0.
\end{aligned}
$$

State Estimation via Backstepping Observer

- Observer Design

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\begin{aligned}
\hat{T}_i(0,t) =& \mathcal{Y}_2(t), \\
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\tilde{T}_i(H(t),t) =& 0.
\end{aligned}
$$

Task : Derive $p(x,t)$ to achieve $\tilde{T} \to 0$ quickly.

State Estimation via Backstepping Observer

- Backstepping Transformation

$$
\begin{aligned}
w(x,t) =& \tilde{T}_i(x,t) - \int_x^{H(t)} \nu(x,y)\tilde{T}_i(y,t)dy, \\
\tilde{T}_i(x,t) =& w(x,t) - \int_x^{H(t)} n(x,y)w(y,t)dy,
\end{aligned}
$$

State Estimation via Backstepping Observer

- Backstepping Transformation

$$w(x,t) = \tilde{T}_i(x,t) - \int_x^{H(t)} \nu(x,y)\tilde{T}_i(y,t)dy,$$

$$\tilde{T}_i(x,t) = w(x,t) - \int_x^{H(t)} n(x,y)w(y,t)dy,$$

- Target System

$$w(0,t) = 0,$$

$$\frac{\partial w}{\partial t}(x,t) = D_i \frac{\partial^2 w}{\partial x^2}(x,t) - \lambda w(x,t),$$

$$w(H(t),t) = 0.$$

State Estimation via Backstepping Observer

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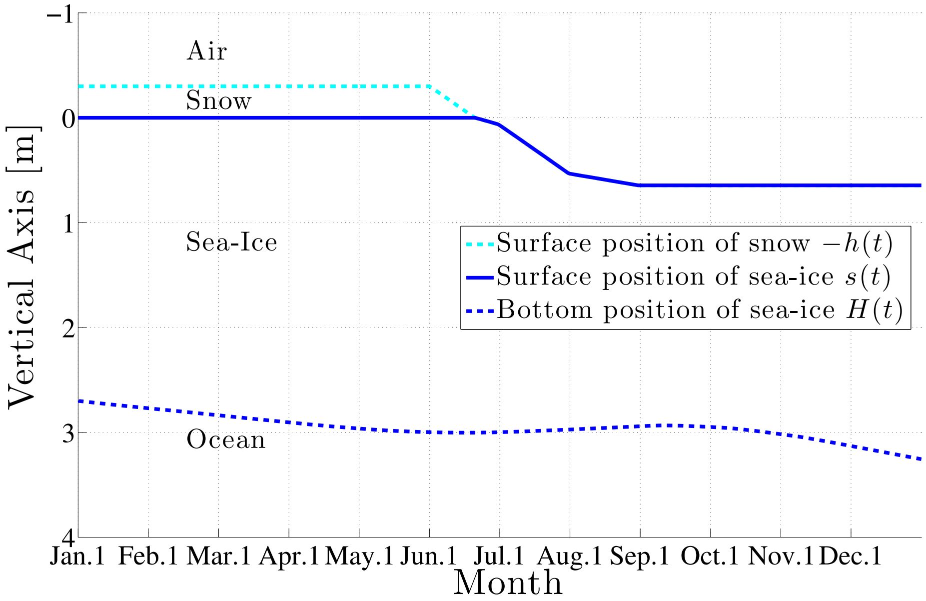
$$w(H(t),t) = 0.$$

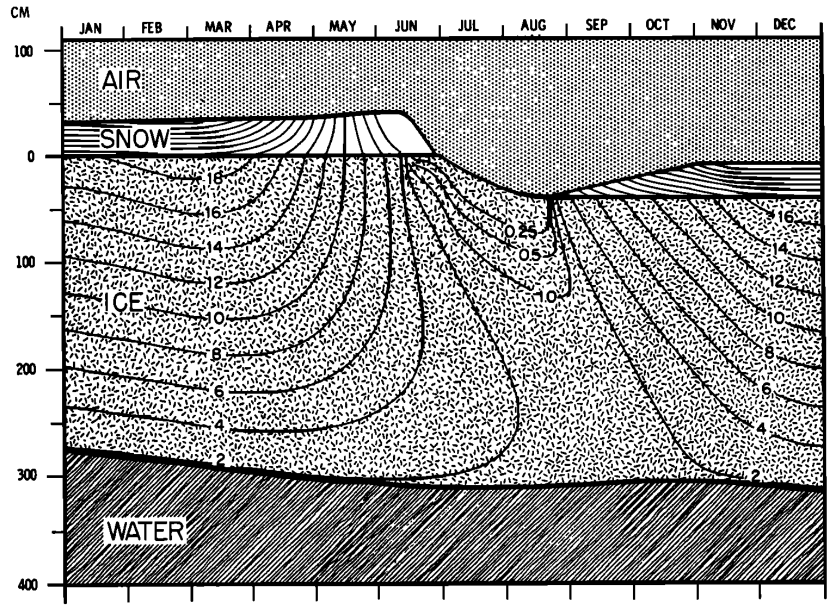
- Gain Derivation

$$p(x,t) = -\lambda x \frac{I_1\left(\sqrt{\frac{\lambda}{D_i}(H(t)^2 - x^2)}\right)}{\sqrt{\frac{\lambda}{D_i}(H(t)^2 - x^2)}},$$

Online Calculation

Simulation Test of MU71





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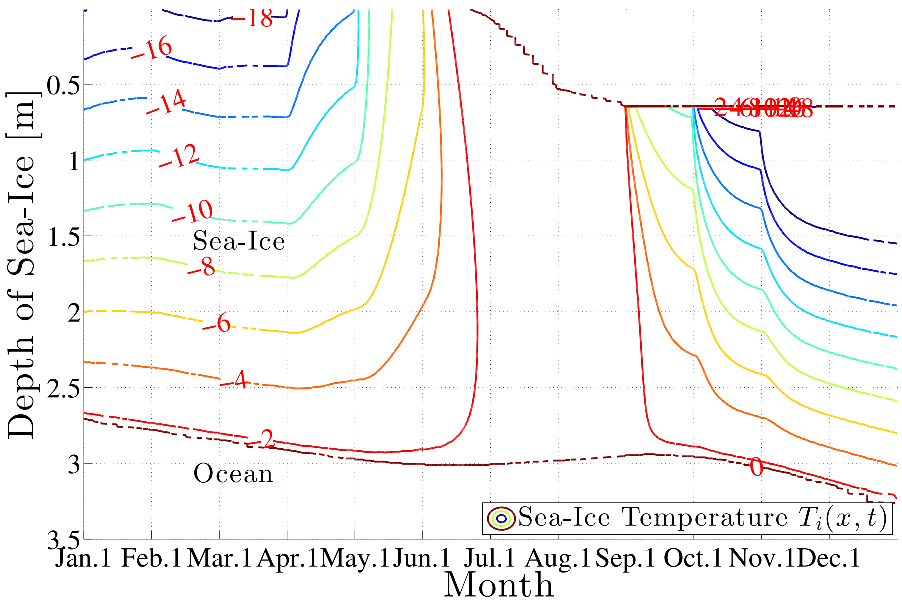
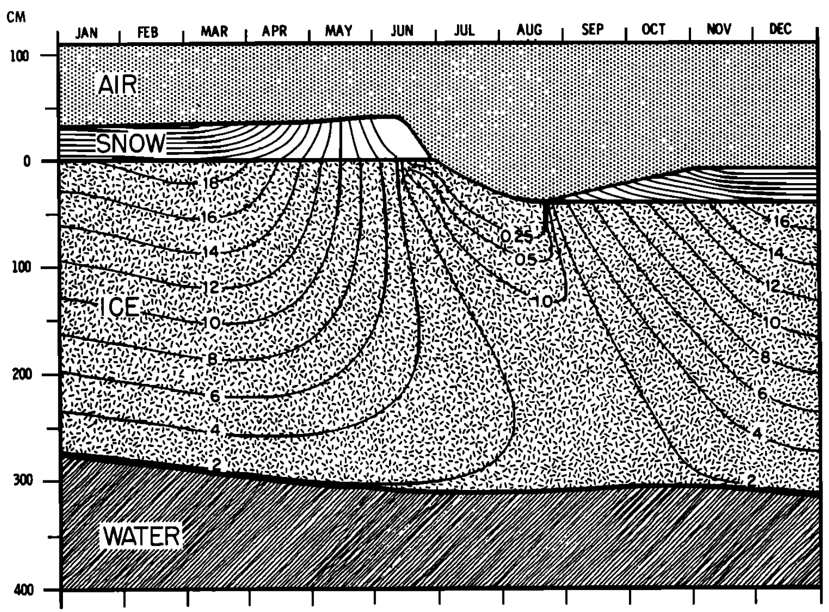
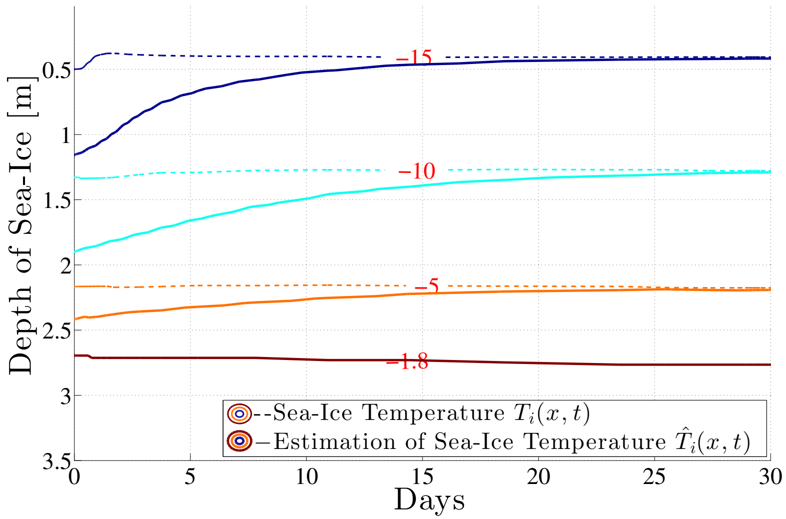


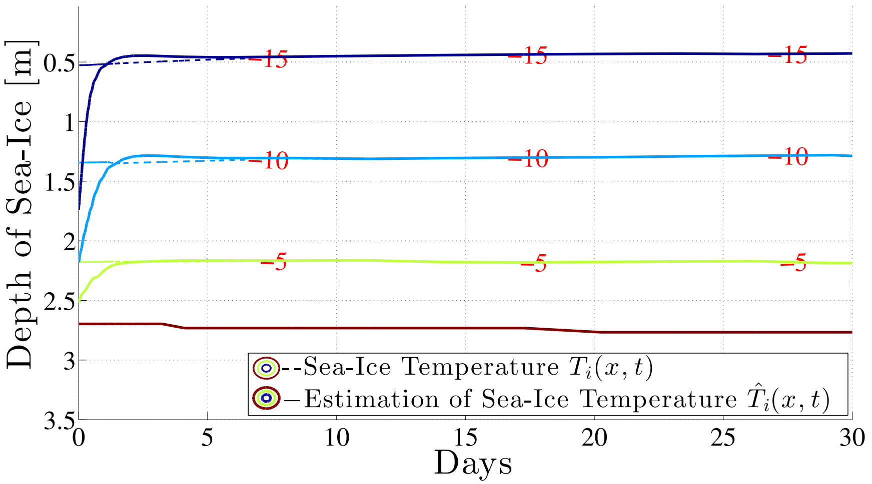


Simulation of Temperature Estimation

- Open-Loop Estimation



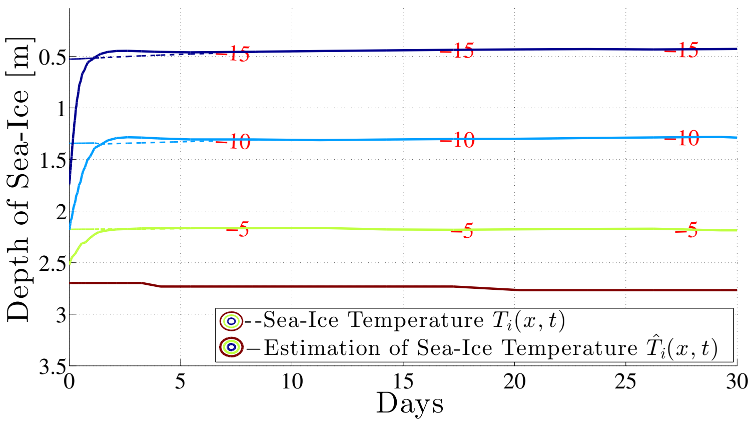
- Backstepping Observer



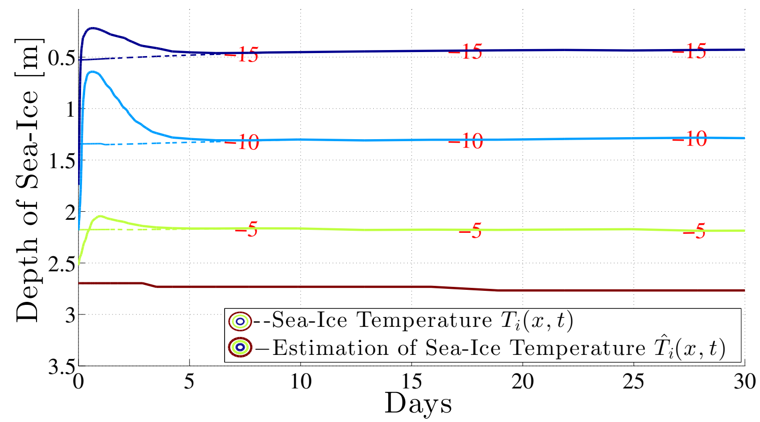
Faster Convergence

Gain Tuning

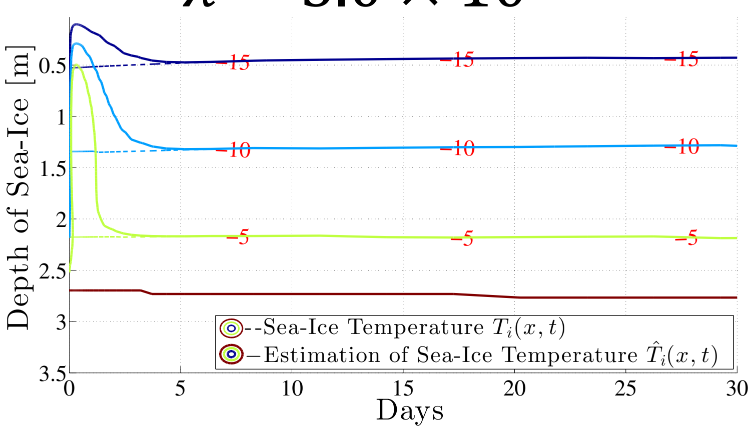
$\lambda = 0.5 \times 10^{-5}$



$\lambda = 1.0 \times 10^{-5}$



$\lambda = 5.0 \times 10^{-5}$



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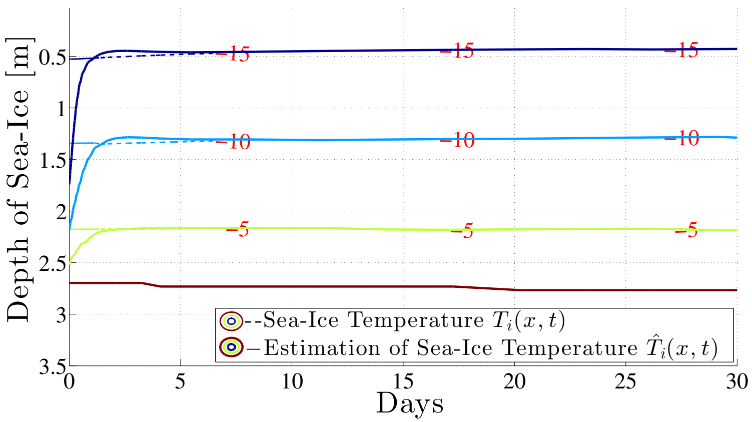

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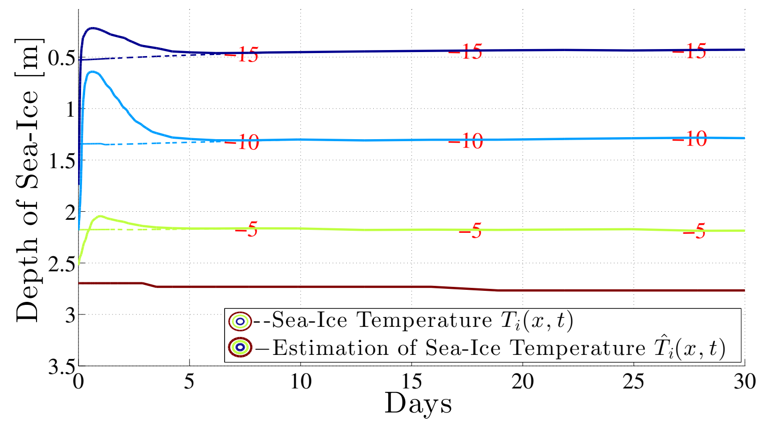

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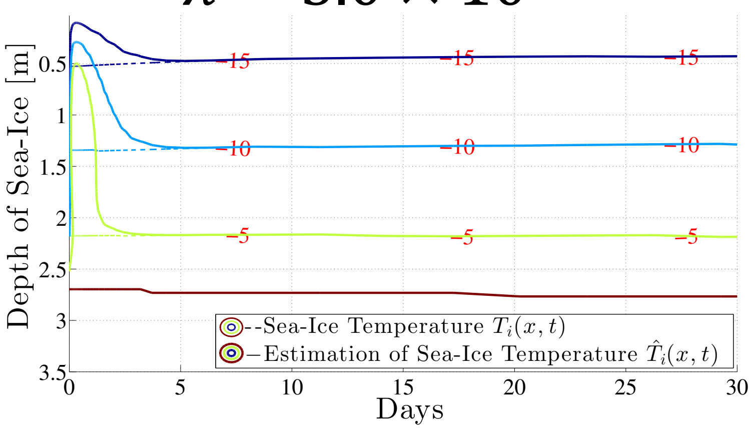

Gain tuning shows the tradeoff between convergence speed and overshoot

Future Work

- Observer design with less measurements
- Comparison with a well-known estimator (e.g. Kalman filter)
- Implementation using empirical data.

Acknowledgment

Prof. I. Eisenman

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Dr. I. Fenty

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