Arctic Sea Ice Temperature Profile Estimation via Backstepping Observer Design

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What Is Sea Ice?

 Sea ice is frozen ocean water.
 (while icebergs, glaciers, etc. originate in land)



• It covers 12% of the ocean.

Why Is Arctic Sea Ice Important?

• Affects global climate by reflection of solar energy.



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Ice-albedo positive feedback

Why Is Arctic Sea Ice Important?

• Recent decline of Arctic sea ice



Sea Ice in Global Climate Model



Sea Ice in Global Climate Model



• Time-evolution of thickness distribution by Thorndike, et al, 1975

Sea Ice in Global Climate Model



 $(T_s(x,t),T_i(x,t))\cdots$ Temp. of snow, sea ice













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Simulation Result by MU71



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Comparison with Empirical Data



Empirical Data (Untersteiner 1969)



Comparison with Empirical Data



Empirical Data (Untersteiner 1969)



Problem Statement

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- 1) Recent data shows no annual cycle.
- 2) Complete profile of sea ice temperature is hard to measure.

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• **Our Goal** <u>Estimate the temperature profile</u> via available measurements.

- Method
- 1) Design an estimator for simplified MU71 theoretically.
- 2) Apply the estimator to original MU71 numerically.

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• Observer Design

$$\begin{split} \hat{T}_{i}(0,t) &= \mathcal{Y}_{2}(t), \\ \frac{\partial \hat{T}_{i}}{\partial t}(x,t) &= D_{i} \frac{\partial^{2} \hat{T}_{i}}{\partial x^{2}}(x,t) + \bar{I}_{0} \kappa_{i} e^{-\kappa_{i} x} \\ &+ p(x,t) \left(\mathcal{Y}_{3}(t) - \frac{\partial \hat{T}_{i}}{\partial x}(\mathcal{Y}_{1}(t),t) \right), \quad 0 < x < \mathcal{Y}_{1}(t) \end{split}$$

 $\hat{T}_i(\mathcal{Y}_1(t), t) = T_{m2}.$

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• Error System

$$\begin{split} \tilde{T}_i(0,t) =& 0, \\ \frac{\partial \tilde{T}_i}{\partial t}(x,t) =& D_i \frac{\partial^2 \tilde{T}_i}{\partial x^2}(x,t) - p(x,t) \frac{\partial \tilde{T}_i}{\partial x}(H(t),t) \\ \tilde{T}_i(H(t),t) =& 0. \end{split}$$

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Task : Derive p(x, t) to achieve $\tilde{T} \to 0$ quickly.

• Backstepping Transformation

$$w(x,t) = \tilde{T}_i(x,t) - \int_x^{H(t)} \nu(x,y) \tilde{T}_i(y,t) dy,$$
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$$\begin{split} w(0,t) =& 0, \\ \frac{\partial w}{\partial t}(x,t) =& D_i \frac{\partial^2 w}{\partial x^2}(x,t) - \lambda w(x,t), \\ w(H(t),t) =& 0. \end{split}$$

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• Gain Derivation

$$p(x,t) = -\lambda x \frac{I_1\left(\sqrt{\frac{\lambda}{D_i}(\boldsymbol{H}(t)^2 - x^2)}\right)}{\sqrt{\frac{\lambda}{D_i}(\boldsymbol{H}(t)^2 - x^2)}},$$

Online Calculation

Simulation Test of MU71



Simulation Test of MU71



Simulation of Temperature Estimation





Gain Tuning



Gain Tuning



 $\lambda = 1.0 \times 10^{-5}$

Gain tuning shows the tradeoff between <u>convergence</u> <u>speed</u> and <u>overshoot</u>

Future Work

• Observer design with less measurements

• Comparison with a well-known estimator (e.g. Kalman filter)

• Implementation using empirical data.

Acknowledgment



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